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Assignment No. - 2

Q1 State the formula radius of curvature for parametric curves.

Solve Radius of curvature for parametric curves :-

$$x = f(t) \quad , \quad y = g(t)$$

$$x = f(r,\theta) \leftarrow (x = r\cos\theta , y = r\sin\theta) \Rightarrow y = g(r,\theta)$$

$$x^2 + y^2 = r^2 \Rightarrow (\cos^2 + \sin^2\theta) = r^2$$

$$y^2 = 4ax$$

$$(at^2 , 2at)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left(\frac{y'}{x'}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \Rightarrow \frac{d}{dx}\left(\frac{y'}{x'}\right) \Rightarrow \frac{d}{dt}\left(\frac{y'}{x'}\right)\frac{dt}{dx}$$

let, $\frac{dx}{dt} = x'$ & $\frac{dt}{dx} = \frac{1}{x'}$

$$\frac{x'y'' - y'x''}{(x')^2} \cdot \frac{1}{x'}.$$

$$\frac{x'y'' - y'x''}{(x')^3} = \frac{d^2y}{dx^2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\rho = \frac{\left[1 + \left(\frac{y'}{x'}\right)^2\right]^{3/2}}{\frac{x'y'' - y'x''}{(x')^3}}$$

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$$\rho = \frac{\left[\dfrac{x'^2 + y'^2}{x'^2}\right]^{3/2} \cdot x'^3}{x'y'' - y'x''}$$

$$\rho = \frac{\dfrac{(x'^2 + y'^2)^{3/2} \cdot \cancel{x'^3}}{\cancel{(x')^{2 \cdot \frac{3}{2}}}}}{x'y'' - y'x''}$$

$$\boxed{\rho = \frac{(x'^2 + y'^2)^{3/2}}{x'y'' - y'x''}}$$

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Saathi

Q2 Find the Asymptotes of the following curves: $x^3+yx^2-xy^2-y^3-3x-y-1=0$

Solu: Let $x=1$, $y=m$

$\phi_3(m) = 1+m-m^2-m^3$

$\phi_1(m) = -3-m \Rightarrow \phi_3(m)=0$

$-m^3-m^2+m+1=0$

$m=1$

$m-1=0$

$(m-1)(-m^2-2m-1)=0$

$-(m-1)(m^2+2m+1)=0$

$m^2+2m+1)=0$

$m^2+m+m+1=0$

$m(m+1).1(m+1)=0$

$(m+1)(m+1)$

$(m+1)^2=0$

$m = -1,-1$

$m=1,-1,-1$

$\frac{c^2}{2!}\phi_3''(-1)+\frac{c}{1!}\phi_3'(-1)+\phi_1(-1)=0$

$\phi_3'(m) = 1-2m-3m^2$

$= -2-6m$

$\phi_3''(m) = -2+6=4$

$\phi_1(-1) = -2$

$\frac{c^2}{2}(4)+0-2=0$

$2c^2-2=0 \Rightarrow 2c^2=2 \Rightarrow c^2=1$

$c=\pm 1$

for $m=-1$

$y=-x+1 \Rightarrow y=-x-1.$

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Q3 Calculate the radius of curvature of the curve $y = a \sinh 2x$.

Solve Formula :- $$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$$

Given :- $a \sinh 2x$

$$\frac{dy}{dx} = a \cosh 2x \cdot 2$$

$$= 2(a \cosh 2x)$$

$$\frac{d^2y}{dx^2} = 2(-a \sinh 2x) \cdot 2$$

$$= -4 (a \sinh 2x)$$

$$\rho = \frac{\left[1 + \{2 (a \cosh 2x)\}^2\right]^{3/2}}{-4 (a \sinh 2x)}$$

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Q4 If $f(x) = 2x^3 - 9x^2 + 12x + 6$ Then execute Maximum and minimum value in $[0, 2]$.

Solve

$f(x) = 2x^3 - 9x^2 + 12x + 6$

$f'(x) = 6x^2 - 18x + 12$

$f'(x) = 0$

$6x^2 - 18x + 12 = 0$

$6(x^2 - 3x + 2) = 0$

$(x-1)(x-2) = 0$

$x = 1, x = 2$

$f''(x) = 12x - 18$

$f''(1) = 12 \times 1 - 18 = -6 < 0$ at $x = 1$ Maxima

$f''(2) = 12 \times 2 - 18 = 6 > 0$ at $x = 2$ Minima

$f(1) = 2(1)^3 - 9(1)^2 + 12(1) + 6 = 2 - 9 + 12 + 6 = 11$

$f(2) = 2(2)^3 - 9(2)^2 + 12(2) + 6 = 16 - 36 + 24 + 6 = 10$

$f(0) = 2(0)^3 - 9(0)^2 + (12)(0) + 6 = 6$

Maxima at $x = 1$ i.e. value $= 11$

Minima at $x = 0$ i.e. value $= 6$

Q5 State and Prove the leibnitz theorem.

Solve If you have two functions $u(x)$ & $v(x)$ both of which are differentiable w.r. to x, then derivates of their product, $u(x).v(x)$ is given by

$$(uv)' = u'v + uv'$$

Now, let's prove this theorem.

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$$(uv)' = \lim_{h \to 0} \frac{u(x+h) \cdot v(x+h) - u(x) \cdot v(x)}{h}$$

$$(uv)' = \lim_{h \to 0} \frac{[u(x+h) v(x+h) - u(x) v(x)]}{h}$$

$$\lim_{h \to 0} \frac{[u(x+h) v(x+h) - u(x) v(x+h)]}{h}$$

$$+ \lim_{h \to 0} \frac{[u(x) v(x+h) - u(x)]}{h}$$

$$\lim_{h \to 0} \frac{[u(x+h) - u(x)] v(x+h) + u(x)]}{h}$$

$$\lim_{h \to 0} \frac{[v(x+h) - v(x)]}{h}$$

$$(uv)' = u' v^* + u v'$$

Hence

proved.